

Convolution model for the structure functions of the nucleon

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Abstract

We start from an MIT-bag model calculation which provides information about the constituent quark distributions in the nucleon. The constituent quarks, however, are themselves considered as complex objects whose partonic substructure is resolved in deep inelastic scattering. This gives rise to structure functions of the constituent quarks which, in the unpolarized case, are fitted to data at a fixed scale employing three model parameters. Using Q^2 -evolution equations the data are also well described at other scales. For the spindependent structure functions $g_1^{p,n}$ we additionally have to introduce polarization functions for valence and sea quarks which are determined by exploiting the x -dependence of the available proton data only. A negatively polarized sea in the range $x \geq 0.01$ is suggested. We are then capable of predicting the shape of the neutron structure function g_1^n which turns out to be in good agreement with experiment. Finally we present an estimate for the transversely polarized structure function g_2 , offering the possibility of extracting the twist-3 contribution and rating its importance.

1 Introduction

During the last two decades the extensive performance of high energy experiments has made an important contribution to our understanding of the nucleon substructure. In particular deep inelastic scattering (DIS) of leptons from nucleon targets enables the determination of the nucleon structure functions which contain basic information about the quark and gluon dynamics in hadronic matter. In order to study this dynamics on the theoretical side, one should actually exploit QCD as the underlying theory of strong interaction physics which, however, is hardly understood in the nonperturbative domain, thus offering no possibility of calculating structure functions from first principles. For this reason one is forced to utilize phenomenological approaches. But even then we face the problem that all available models are formulated in terms of few quark degrees of freedom and therefore tailored to reproduce the low energy properties of the nucleon, whereas for the evaluation of realistic structure functions the nucleon wave function should reflect the dynamics of a large number of pointlike partons resolved in DIS. Accordingly the structure functions obtained in such models have to be associated with a low momentum scale and one has to think about procedures to bring them in relation to reality, i.e. the experimental situation at $Q_{\text{exp}}^2 \gg 1 \text{ GeV}^2$.

In this work we employ the traditional MIT-bag model [1] to explicitly calculate structure functions in the low- Q^2 domain [2, 3, 4]. The picture we then have in mind is to regard the bag quarks as effective objects, in the subsequent sections called *constituent quarks*, which reveal their complicated substructure in DIS. This gives rise to distribution functions characterizing the dynamics of the quark partons and gluons building up these effective quarks [5]. Later in the discussion these distribution functions are denoted by ϕ_v, ϕ_s and ϕ_g . While the constituent quark distributions obtained in the bag model provide the nonperturbative input of the structure functions, the parton distributions inside the constituent quarks are connected with large Q^2 as involved in DIS. In this sense the constituent quarks can be viewed as a bridge between the nonperturbative and the perturbative regime, leading to the convolution model for the substructure of the nucleon [6].

The first goal of our investigations is to demonstrate that a satisfactory and consistent description of the unpolarized structure functions can be achieved in such a convolution model approach. In this framework the Q^2 -evolution equations can

be completely expressed in terms of the parton distribution functions ϕ , exposing the change of the substructure of the constituent quarks with increasing resolution. We are then going to apply the formalism to the longitudinally polarized structure functions of the nucleon, i.e. $g_1^{p,n}$, where a careful analysis of the latest data [7, 8] is presented. In doing this, we solely exploit the x -dependence of g_1^p and use neither the extrapolated integrals over the data nor information from hyperon β -decays. Our special attention in this analysis is given to the polarization of the sea quarks which turns out to be essential for the understanding of g_1 . With the sea polarization in hands a prediction for the neutron structure function g_1^n can be made and compared with the available data [9, 10]. Finally we will also present an estimate for the structure function g_2 which will be experimentally accessible via transversely polarized scattering events [11, 12] and for the first time offers the possibility of extracting higher-twist contributions in DIS [13].

The paper is organized as follows. In Sect. 2 we will start with a brief review of the DIS formalism and the definitions of structure functions. Sect. 3 deals with the interpretation of structure functions resulting from quark model calculations. This leads to the concept of the convolution model, as explained in Sect. 4. Thereupon we will successively devote us to the discussion of F_2 , g_1 , and g_2 , corresponding to Sects. 5, 6, and 7, respectively. In Sect. 8 we give a conclusion of our results as well as an outlook on problems which remain to be solved.

2 Deep inelastic scattering and structure functions

The Feynman diagram which describes the deep inelastic lepton–nucleon scattering process $\ell + N \rightarrow \ell' + X$ in leading order of the electromagnetic interaction is depicted in Fig. 1. For further considerations we go into the target rest frame in which

$$p = (M, 0) , \quad k = (E, \mathbf{k}) , \quad k' = (E', \mathbf{k}') \quad (1)$$

holds and write the frequently appearing quantities as

$$\nu = \frac{p \cdot q}{M} = E - E' , \quad (2)$$

$$Q^2 = -q^2 , \quad (3)$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M\nu} . \quad (4)$$

Employing the usual Feynman rules to evaluate the inclusive differential cross section, one finds [14]

$$\frac{d^2\sigma}{dE' d\Omega_\ell} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L^{\mu\nu} W_{\mu\nu} . \quad (5)$$

While the leptonic tensor $L^{\mu\nu}$ can be calculated explicitly due to the pointlike nature of the lepton, the hadronic tensor $W_{\mu\nu}$ contains all the relevant information about the substructure of the nucleon and therefore represents a much more complicated object. For the spin averaged case we get two completely symmetric tensors [15]

$$\begin{aligned} L_S^{\mu\nu} &= \frac{1}{2} \sum_{s,s'} [\bar{u}(k', s') \gamma^\mu u(k, s)] [\bar{u}(k', s') \gamma^\nu u(k, s)] \\ &= \frac{1}{2} \text{Tr} [\gamma^\mu (\not{k} + m) \gamma^\nu (\not{k}' + m)] , \end{aligned} \quad (6)$$

$$W_{\mu\nu}^S = \frac{1}{4\pi M} \frac{1}{2} \sum_\sigma \int d^4\xi e^{iq \cdot \xi} \langle p, \sigma | [J_\mu(\xi), J_\nu(0)]^S | p, \sigma \rangle , \quad (7)$$

J_μ being the operator of the hadronic electromagnetic current. The unpolarized structure functions $W_{1,2}$ are defined by means of the most general symmetric tensor respecting current conservation and Lorentz invariance [14]

$$W_{\mu\nu}^S = W_1(\nu, q^2) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2(\nu, q^2)}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) . \quad (8)$$

At that point one usually switches over to dimensionless structure functions $F_{1,2}$ which are related to $W_{1,2}$ in the following way:

$$F_1(x, Q^2) = M W_1(\nu, q^2) , \quad (9)$$

$$F_2(x, Q^2) = \nu W_2(\nu, q^2) . \quad (10)$$

In the naive quark parton model [16] these functions can be expressed in terms of quark distribution functions q_i^N of the various flavours i in the nucleon N ,

$$F_2^N(x) = 2x F_1^N(x) = \sum_{i=1}^{2n_f} e_i^2 x q_i^N(x) , \quad (11)$$

where the independence on Q^2 is a consequence of the neglect of QCD-corrections to the dominant photon-parton scattering process.

In the case of spinindependent lepton-nucleon scattering the information which is specific for the polarization effects enters the completely antisymmetric parts of the

tensors [14, 15]

$$L_A^{\mu\nu} = \frac{1}{2} \text{Tr} \left[\gamma^\mu \gamma^5 (\not{k} + m) \gamma^\nu (\not{k}' + m) \right] , \quad (12)$$

$$W_{\mu\nu}^A = \frac{1}{4\pi M} \int d^4\xi e^{iq \cdot \xi} \langle p, \sigma | [J_\mu(\xi), J_\nu(0)]^A | p, \sigma \rangle \quad (13)$$

$$= i \varepsilon_{\mu\nu\alpha\rho} q^\alpha \left[s^\rho M G_1 + \left(s^\rho \frac{p \cdot q}{M} - p^\rho \frac{s \cdot q}{M} \right) G_2 \right] , \quad (14)$$

with s^ρ denoting the spin vector of the nucleon. The last expression again constitutes the most general ansatz and thus serves as a definition for the functions $G_{1,2}$ which are related to the commonly used structure functions $g_{1,2}$ by

$$g_1(x, Q^2) = M^2 \nu G_1(\nu, q^2) , \quad (15)$$

$$g_2(x, Q^2) = M \nu^2 G_2(\nu, q^2) . \quad (16)$$

In the naive parton model one finds [16]

$$g_1^N(x) = \frac{1}{2} \sum_{i=1}^{2n_f} e_i^2 \left(q_i^{\uparrow N}(x) - q_i^{\downarrow N}(x) \right) \equiv \frac{1}{2} \sum_{i=1}^{2n_f} e_i^2 \Delta q_i^N(x), \quad (17)$$

$$g_T^N(x) \equiv g_1^N(x) + g_2^N(x) = \frac{1}{2} \sum_{i=1}^{2n_f} e_i^2 \left(\tilde{q}_i^{\uparrow N}(x) - \tilde{q}_i^{\downarrow N}(x) \right) \equiv \frac{1}{2} \sum_{i=1}^{2n_f} e_i^2 \Delta \tilde{q}_i^N(x), \quad (18)$$

where $q_i^{\uparrow(\downarrow)N}$ characterize the probabilities to find in a longitudinally polarized nucleon a quark with flavour i and spin alignment parallel (antiparallel) to the nucleon spin, while $\tilde{q}_i^{\uparrow(\downarrow)N}$ are the corresponding probabilities for transversely polarized nucleons.

In addition to the fundamental process $\gamma^* q \rightarrow q$ the framework of QCD supplies further processes which lead to corrections of the naive parton model. Taking such modifications into account, the above introduced quark distributions acquire a Q^2 -dependence. Furthermore the QCD-improved formalism also involves the gluon distribution $g(x, Q^2)$. Adopting the variable

$$\kappa = \frac{2}{\beta_0} \ln \left(\frac{\ln \left(\frac{Q_0^2}{\Lambda^2} \right)}{\ln \left(\frac{Q^2}{\Lambda^2} \right)} \right) \equiv -\frac{2}{\beta_0} \ln \zeta , \quad (19)$$

with $\beta_0 = 11 - \frac{2}{3}n_f$, and using the non-singlet (NS) and singlet (S) quark distributions

$$q^{\text{NS}}(x, Q^2) = \sum_{i=1}^{n_f} \left(q_i(x, Q^2) - \bar{q}_i(x, Q^2) \right) , \quad (20)$$

$$q^{\text{S}}(x, Q^2) = \sum_{i=1}^{n_f} \left(q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right) , \quad (21)$$

the leading order QCD–evolution equations finally can be written as [17]

$$\frac{d}{d\kappa} q^{\text{NS}}(x, Q^2) = \int_x^1 \frac{dy}{y} P_{q \rightarrow qg}\left(\frac{x}{y}\right) q^{\text{NS}}(y, Q^2), \quad (22)$$

$$\frac{d}{d\kappa} \begin{pmatrix} q^{\text{S}}(x, Q^2) \\ g(x, Q^2) \end{pmatrix} = \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{q \rightarrow qg}\left(\frac{x}{y}\right) & 2n_f P_{g \rightarrow q\bar{q}}\left(\frac{x}{y}\right) \\ P_{q \rightarrow gq}\left(\frac{x}{y}\right) & P_{g \rightarrow gg}\left(\frac{x}{y}\right) \end{pmatrix} \begin{pmatrix} q^{\text{S}}(y, Q^2) \\ g(y, Q^2) \end{pmatrix}, \quad (23)$$

P being the splitting functions of the various splitting processes. These evolution equations enable the determination of F_2 at any perturbatively accessible scale Q^2 , provided one knows the function at a reference scale Q_0^2 . Whereas on the one hand for the longitudinally polarized distribution functions analogous equations hold with q and g replaced by Δq and Δg , respectively, a careful analysis of the operator structure of g_2 [18] on the other hand reveals that g_2 also receives twist–3 contributions so that its Q^2 –evolution remains a yet unsolved problem.

3 Interpretation of quark model calculations

So far the employment of the parton model merely resulted in a parametrization of the structure functions in terms of quark distributions which contain nonperturbative information about the hadronic bound state. From the theoretical point of view it must be the goal to calculate the structure functions, and with those also the quark distributions, using appropriate models of the nucleon. Basically this is possible with the help of Eqs. (7),(8) and (13),(14) after having performed an operator product expansion of the current commutator. Equipped with this formalism the final step towards the explicit calculation of structure functions consists in inserting a nucleon wave function into the matrix elements of (7) and (13). In order to get realistic results, this wave function should be formulated in terms of pointlike partons which are resolved in DIS. In practice, however, there is so far no knowledge whatsoever how to transform the dynamics of the elementary constituents of the nucleon into a wave function. This is why one is for the presence forced to utilize well established, phenomenological low energy models of the nucleon which are based on few quark degrees of freedom. One of these models is the MIT-bag model [1] which is formulated relativistically and incorporates confinement in a simple and transparent manner, thus being very well suited for the calculation of structure functions. Using the well known $SU(6)$ nucleon wave functions, the results for the physical structure

functions in the bag model read as follows [2, 3, 4]

$$\begin{aligned} \left(F_1^p(x) \right)_{\text{bag}} = & \frac{MR}{2\pi} \frac{\varepsilon^4}{\varepsilon^2 - \sin^2 \varepsilon} \left\{ \int_{|MRx-\varepsilon|}^{\infty} d\beta \beta \left[T_0^2(\varepsilon, \beta) + T_1^2(\varepsilon, \beta) \right. \right. \\ & \left. \left. - \frac{2}{\beta} (\varepsilon - MRx) T_0(\varepsilon, \beta) T_1(\varepsilon, \beta) \right] + (x \rightarrow -x) \right\}, \quad (24) \end{aligned}$$

$$\begin{aligned} \left(g_1^p(x) \right)_{\text{bag}} = & \frac{5}{9} \frac{MR}{2\pi} \frac{\varepsilon^4}{\varepsilon^2 - \sin^2 \varepsilon} \left\{ \int_{|MRx-\varepsilon|}^{\infty} d\beta \beta \left[T_0^2(\varepsilon, \beta) \right. \right. \\ & + \left(2 \left(\frac{\varepsilon - MRx}{\beta} \right)^2 - 1 \right) T_1^2(\varepsilon, \beta) \\ & \left. \left. - \frac{2}{\beta} (\varepsilon - MRx) T_0(\varepsilon, \beta) T_1(\varepsilon, \beta) \right] + (x \rightarrow -x) \right\}, \quad (25) \end{aligned}$$

$$\begin{aligned} \left(g_T^p(x) \right)_{\text{bag}} = & \frac{5}{9} \frac{MR}{2\pi} \frac{\varepsilon^4}{\varepsilon^2 - \sin^2 \varepsilon} \left\{ \int_{|MRx-\varepsilon|}^{\infty} d\beta \beta \left[T_0^2(\varepsilon, \beta) \right. \right. \\ & \left. \left. - \left(\frac{\varepsilon - MRx}{\beta} \right)^2 T_1^2(\varepsilon, \beta) \right] + (x \rightarrow -x) \right\}, \quad (26) \end{aligned}$$

with

$$T_n(\varepsilon, \beta) = \int_0^1 dz z^2 j_n(\varepsilon z) j_n(\beta z). \quad (27)$$

In the case of the neutron we have $(F_1^n(x))_{\text{bag}} = \frac{2}{3} (F_1^p(x))_{\text{bag}}$, whereas both $(g_1^n(x))_{\text{bag}}$ and $(g_T^n(x))_{\text{bag}}$ vanish identically. The quantities ε and MR entering the expressions for the structure functions denote the lowest bag frequency and the product mass times bag radius, respectively, and are completely fixed by the boundary and stability condition of the bag, giving $\varepsilon = 2.04$ and $MR = 4\varepsilon$ [19], so that Eqs. (24)–(26) do not contain any free parameter. As a consequence of broken translational invariance in the static approximation of the bag model the structure functions receive nonvanishing support from the unphysical region $x > 1$. For this reason modified versions of the bag model [20, 21] are equipped with a Peierls–Yoccoz projection [22] onto momentum eigenstates which weakens but not completely cures the support problem. In this work we will disregard such a projection since we consider the latter as a minor deficiency. In fact, the numerical contribution from the region $x > 1$ is very small and the normalizations

$$\int_0^1 dx \left(F_1^p(x) \right)_{\text{bag}} = \frac{1}{2}, \quad (28)$$

$$\int_0^1 dx \left(g_1^p(x) \right)_{\text{bag}} = \frac{1}{6} \left(\frac{g_A}{g_V} \right)_{\text{bag}} , \quad (29)$$

$$\int_0^1 dx \left(g_2^p(x) \right)_{\text{bag}} = 0 \quad (30)$$

are fulfilled in good approximation. Here $(g_A/g_V)_{\text{bag}} = 1.09$ denotes the ratio of the weak coupling constants in the bag model which underestimates the experimental one by roughly 12% [19].

The results for the structure functions summarized in the previous paragraph were obtained with the help of a wave function which is formulated in terms of three bag quarks. The dynamics of these degrees of freedom yields a fair description of the low energy properties of the nucleon which are characterized by observables such as mass, magnetic moment, weak coupling constants, charge radius, and electromagnetic form factors. All these quantities are associated with a very low momentum scale $Q^2 \ll 1 \text{ GeV}^2$. Comparing, however, the structure functions derived from the nucleon wave function of the bag model with experiment, *no* agreement can be seen. As already mentioned before, this of course is due to the fact that the structure functions measured in DIS are associated with very large momentum scales $Q^2 \gg 1 \text{ GeV}^2$ implying a high resolution of the nucleon substructure. In order to bring then the structure functions of the bag model in relation to reality, we regard the quarks of the bag model as effective degrees of freedom. To be precise, we understand these effective quarks as clusters which in addition to the valence quarks also consist of sea quarks and gluons [6]. The effective quarks defined in this sense are designated as *constituent quarks* in the following. Accordingly the structure functions calculated in the framework of the bag model, Eqs. (24)–(26), can be interpreted as linear combinations of constituent quark distributions, labeled by the index C . In analogy to Eqs. (11), (17) and (18) we then get

$$\frac{1}{2x} \left(F_2^N(x) \right)_{\text{bag}} = \left(F_1^N(x) \right)_{\text{bag}} = \frac{1}{2} \sum_C e_C^2 q_C^N(x) , \quad (31)$$

$$\left(g_1^N(x) \right)_{\text{bag}} = \frac{1}{2} \sum_C e_C^2 \Delta q_C^N(x) , \quad (32)$$

$$\left(g_T^N(x) \right)_{\text{bag}} = \frac{1}{2} \sum_C e_C^2 \Delta \tilde{q}_C^N(x) . \quad (33)$$

4 The convolution model

The concept of the constituent quarks serves as a bridge between the nonperturbative and perturbative regime. At low Q^2 -scales the resolution is very poor and the hadron appears to be a bound state of three constituent quarks which seem to be structureless. In reality, however, they have a complicated substructure which is resolved in DIS. On the formal level this picture can be translated into a corresponding diagram [23] depicted in Fig. 2. It describes a nucleon N with momentum p containing a constituent quark C with momentum k which in turn consists of a quark parton of flavour i carrying momentum \tilde{k} . This parton absorbs the virtual photon. Fig. 2 is a reflection of an impulse approximation which implies the assumptions that final state interactions on the one hand between the fragments of the nucleon and the constituent quark and on the other hand between the fragments of the constituent quark and the parton are neglected. While the latter are suppressed by $1/Q^2$, as in the parton model, there is, however, no argument for the suppression of the former because of the absence of a characteristic mass scale. Nevertheless, in the weak coupling limit of the constituent quarks, as realized in the bag model, the impulse approximation should be justified very well [23].

Taking the momentum dependences of the quark distributions, as given in Fig. 2, into consideration, the probability of finding a quark i in the nucleon N is determined by

$$q_i^N(p, q) = \sum_C \int d^4k q_C^N(p, k) q_i^C(k, q) . \quad (34)$$

After transition to the usual invariants one finally arrives at [23]

$$q_i^N(x, Q^2) = \sum_C \int_x^1 \frac{dy}{y} q_C^N(y) q_i^C\left(\frac{x}{y}, Q^2\right) . \quad (35)$$

This is the fundamental relation of the convolution model which describes the parton distributions in the nucleon as a convolution of the constituent quark distributions in the nucleon and the parton distributions in the constituent quarks.

Let us now have a closer look at the parton distributions in the constituent quarks. Since the formalism has to include valence as well as sea quarks, it is advantageous to decompose the functions q_i^C into two corresponding parts [24],

$$q_i^C(z, Q^2) = \phi_v(z, Q^2) \delta_{iC} + \phi_s^{i/C}(z, Q^2) . \quad (36)$$

While the sea quark distributions have to be characterized by an isospin label in general, the valence quark distribution should be isospin independent and has to fulfill the normalization constraint

$$\int_0^1 dz \phi_v(z, Q^2) = 1. \quad (37)$$

This together with the δ_{iC} of Eq. (36) is a reflection of associating with each constituent quark just one valence quark of the same flavour. For the sea quark distributions it is reasonable to impose isospin symmetry with respect to the constituent quarks as well as full symmetry of the strange quarks, yielding finally the most general relations

$$\phi_s^{u/U}(z, Q^2) = \phi_s^{\bar{u}/U}(z, Q^2) = \phi_s^{d/D}(z, Q^2) = \phi_s^{\bar{d}/D}(z, Q^2), \quad (38)$$

$$\phi_s^{u/D}(z, Q^2) = \phi_s^{\bar{u}/D}(z, Q^2) = \phi_s^{s/U}(z, Q^2) = \phi_s^{\bar{s}/U}(z, Q^2), \quad (39)$$

$$\phi_s^{s/U}(z, Q^2) = \phi_s^{\bar{s}/U}(z, Q^2) = \phi_s^{s/D}(z, Q^2) = \phi_s^{\bar{s}/D}(z, Q^2). \quad (40)$$

5 The unpolarized structure function F_2

Based on the relations of the previous section we are now capable of formulating the convolution model expressions for the unpolarized structure function F_2 . In addition to the constituent quark distributions q_C^N this expression also involves the parton distributions ϕ . The former reflect the dynamics of constituent quarks inside hadrons and contain information about the confinement mechanism. In the following this nonperturbative part of F_2 will be described by the bag model calculation (24). The latter contain information about the parton dynamics in the constituent quarks. Since, however, there is no way to calculate these functions ϕ at a microscopic level, one has to use suitable parametrizations at a reference scale Q_0^2 . In order to minimize the number of free parameters, we start our considerations by equating (38) and (39), i.e. $\phi_s^{u/U}(z, Q^2) = \phi_s^{u/D}(z, Q^2) \equiv \phi_s(z, Q^2)$ and relating (40) to (38) by the z -independent suppression factor $1/2$ [25], i.e. $\phi_s^{s/U}(z, Q^2) = \frac{1}{2}\phi_s^{u/U}(z, Q^2)$. Bearing the normalization condition (37) in mind, we then choose [5]

$$\phi_v(z, Q_0^2) = \frac{\Gamma(a+3/2)}{\Gamma(1/2)\Gamma(a+1)} \frac{1}{\sqrt{z}} (1-z)^a, \quad (41)$$

$$\phi_s(z, Q_0^2) = A \frac{1}{z} (1-z)^b. \quad (42)$$

These expressions are motivated by Regge arguments and dimensional counting rules. Employing (11), (31), (35), (36) and $SU(6)$ -symmetry, we finally get

$$F_2^{p,n}(x, Q^2) = 2x \int_x^1 \frac{dy}{y} \left(F_1^{p,n}(y) \right)_{\text{bag}} \phi_v\left(\frac{x}{y}, Q^2\right) + \frac{22}{3}x \int_x^1 \frac{dy}{y} \left(F_1^p(y) \right)_{\text{bag}} \phi_s\left(\frac{x}{y}, Q^2\right) . \quad (43)$$

The total of three parameters entering ϕ_v and ϕ_s are fitted to experiment at the reference scale $Q_0^2 = 10 \text{ GeV}^2$, giving $a = -0.17$, $b = 4$, and $A = 0.137$. Fig. 3 shows the result in comparison with the data [26].

With the help of the QCD-evolution equations (22) and (23) we can then determine the structure function at any other scale Q^2 which lies in the perturbatively accessible domain. This evolution also involves the gluon distribution which in the convolution model can be written as

$$g(x, Q^2) = 6 \int_x^1 \frac{dy}{y} \left(F_1^p(x) \right)_{\text{bag}} \phi_g\left(\frac{x}{y}, Q^2\right) . \quad (44)$$

Since the total Q^2 -dependence of F_2 is fully contained in the functions ϕ , it is desirable to derive evolution equations for them. Performing the transition to the moments

$$\begin{pmatrix} M_n^{\phi_v}(Q^2) \\ M_n^{\phi_s}(Q^2) \\ M_n^{\phi_g}(Q^2) \end{pmatrix} = \int_0^1 dz z^{n-1} \begin{pmatrix} \phi_v(z, Q^2) \\ \phi_s(z, Q^2) \\ \phi_g(z, Q^2) \end{pmatrix} , \quad (45)$$

$$\begin{pmatrix} A_n^{\text{NS}} \\ A_n^{gq} \\ A_n^{q\bar{q}} \\ A_n^{gg} \end{pmatrix} = \int_0^1 dz z^{n-1} \begin{pmatrix} P_{q \rightarrow qg}(z) \\ P_{q \rightarrow gq}(z) \\ P_{g \rightarrow q\bar{q}}(z) \\ P_{g \rightarrow gg}(z) \end{pmatrix} , \quad (46)$$

$$A_n^\pm = \frac{1}{2} \left(A_n^{\text{NS}} + A_n^{gg} \pm \sqrt{(A_n^{\text{NS}} - A_n^{gg})^2 + 4 A_n^{gq} 2n_f A_n^{q\bar{q}}} \right) , \quad (47)$$

and taking advantage of the properties of convolution integrals, we finally get the solutions

$$M_n^{\phi_v}(Q^2) = \zeta^{-\frac{2}{\beta_0} A_n^{\text{NS}}} M_n^{\phi_v}(Q_0^2) , \quad (48)$$

$$\begin{aligned} M_n^{\phi_s}(Q^2) = & \frac{1}{5} \left\{ \frac{1}{A_n^- - A_n^+} \left\{ \left[(A_n^- - A_n^{\text{NS}}) \zeta^{-\frac{2}{\beta_0} A_n^+} + (A_n^{\text{NS}} - A_n^+) \zeta^{-\frac{2}{\beta_0} A_n^-} \right] \times \right. \right. \\ & \times \left(M_n^{\phi_v}(Q_0^2) + 5 M_n^{\phi_s}(Q_0^2) \right) \\ & \left. \left. + 2n_f A_n^{q\bar{q}} \left(\zeta^{-\frac{2}{\beta_0} A_n^-} - \zeta^{-\frac{2}{\beta_0} A_n^+} \right) M_n^{\phi_g}(Q_0^2) \right\} - M_n^{\phi_v}(Q_0^2) \right\} . \quad (49) \end{aligned}$$

The variable ζ has been defined in Eq. (19), explicit expressions for the moments A can be found in [17]. For the gluon distribution which also cannot be deduced by means of a microscopic model yet, we use the parametrization

$$\phi_g(z, Q_0^2) = B \frac{1}{z} (1-z)^c, \quad (50)$$

with $B = 1.48$, $c = 2.37$. According to (44) this parameter choice corresponds to $g(x, Q_0^2) = 4.27 \frac{1}{x} (1-x)^8$ which respects the momentum sum rule $\int_0^1 dx x (q^S(x, Q^2) + g(x, Q^2)) = 1$ in conjunction with the singlet quark distribution and turns out to be a reasonable ansatz. Finally the QCD-parameter Λ entering ζ is settled at $\Lambda = 200 \text{ MeV}$.

Employing standard techniques [27], we reconstruct the distribution functions $\phi_v(z, Q^2)$ and $\phi_s(z, Q^2)$ from the evolved moments, Eqs. (48), (49), using twelve and three moments, respectively. These distributions tell us in which way the parton substructure of the constituent quarks changes with the resolution Q^2 . The results for typical scales are displayed in Figs. 4 and 5. From these we can easily determine the Q^2 -dependence of the structure function F_2^p ; the results are compared to some data in Figs. 6 and 7.

After having restricted ourselves to the unpolarized structure function of the proton so far, some remarks about the neutron structure function are in order now. As usual we investigate the difference $F_2^p - F_2^n$ which in the convolution model is given by

$$\begin{aligned} [F_2^p - F_2^n](x, Q^2) &= \frac{2}{3} x \int_x^1 \frac{dy}{y} \left(F_1^p(y) \right)_{\text{bag}} \phi_v\left(\frac{x}{y}, Q^2\right) \\ &\quad - \frac{4}{3} x \int_x^1 \frac{dy}{y} \left(F_1^p(y) \right)_{\text{bag}} \left[\phi_s^{u/D}\left(\frac{x}{y}, Q^2\right) - \phi_s^{u/U}\left(\frac{x}{y}, Q^2\right) \right] \end{aligned} \quad (51)$$

Assuming $\phi_s^{u/U} = \phi_s^{u/D}$, as before, the sea quark contribution drops and we are left solely with the valence part. Fig. 8 shows $F_2^p - F_2^n$ in this case for $Q^2 = 40 \text{ GeV}^2$. For large values of x we observe an underestimation of the data which is closely connected to the limit $F_2^n/F_2^p \rightarrow 2/3$ as $x \rightarrow 1$. This is a well known deficiency of the $SU(6)$ -wave function we used at the bag level. There are attempts to cure this problem [21], but one must introduce at least two additional phenomenological parameters at this point, namely the diquark masses of the intermediate spin singlet and spin triplet states. Since we are not interested in the limit $x \rightarrow 1$ anyhow, we

do not make modifications in this direction. For small x the data are overestimated and we get, due to Eqs. (28) and (37), the value $1/3$ for the Gottfried sum rule $\text{GSR} \equiv \int_0^1 \frac{dx}{x} (F_2^p(x, Q^2) - F_2^n(x, Q^2))$. An excellent description of the low- x data and a reproduction of the experimental GSR [29, 30] can be achieved according to (51) by setting $\phi^{u/U} \neq \phi^{u/D}$. This requires, however, the introduction of two parameters, namely $b^{u/U}$ and $b^{u/D}$, instead of just one parameter b . Since we have not found a *microscopic mechanism* yet which could give rise to a difference between $\phi^{u/U}$ and $\phi^{u/D}$, i.e. an $SU(2)$ -breaking at the level of the partons within the constituent quarks, we will not study the GSR in this context, but defer it to an extended version of the convolution model which in addition to the three constituent quarks also takes into account meson cloud effects.

6 The polarized structure function g_1

The investigations of Sect. 5 revealed that the convolution model approach enables a satisfactory and consistent description of the unpolarized structure functions, both as a function of x and Q^2 . In the following we are going to apply the formalism to the spindependent structure function g_1 which is given by the parton model representation (17). Writing the polarized quark distributions as $\Delta\phi_v = \mathcal{P}_v\phi_v$ and $\Delta\phi_s = \mathcal{P}_s\phi_v$, with

$$\mathcal{P}_{v,s}(z, Q^2) = \frac{\phi_{v,s}^\uparrow(z, Q^2) - \phi_{v,s}^\downarrow(z, Q^2)}{\phi_{v,s}^\uparrow(z, Q^2) + \phi_{v,s}^\downarrow(z, Q^2)}, \quad (52)$$

employing (32) and making use of $SU(6)$ -symmetry at the constituent quark level, we find the convolution model relation

$$\begin{aligned} g_1^{p,n}(x, Q^2) &= \int_x^1 \frac{dy}{y} \left(g_1^{p,n}(y) \right)_{\text{bag}} \mathcal{P}_v\left(\frac{x}{y}, Q^2\right) \phi_v\left(\frac{x}{y}, Q^2\right) \\ &+ \frac{11}{5} \int_x^1 \frac{dy}{y} \left(g_1^p(y) \right)_{\text{bag}} \mathcal{P}_s\left(\frac{x}{y}, Q^2\right) \phi_s\left(\frac{x}{y}, Q^2\right). \end{aligned} \quad (53)$$

This equation also implies the in our opinion reasonable assumption that due to the comparatively small mass of the strange-quarks, $m_s^2/Q^2 \ll 1$, their polarization should not be considerably suppressed compared to the polarization of the u- and d-quark flavours. For reasons of simplicity we assume the same polarization for all three flavours. Nonperturbative effects enter the structure functions again via the

bag model, Eq. (25), the functions ϕ are the same as before and the spindependent information is fully contained in the polarization functions \mathcal{P}_v and \mathcal{P}_s .

The particular form of (53) is perfectly appropriate to perform a careful analysis of the latest data. First of all we have a look at \mathcal{P}_v . Imposing the fundamental Bjorken sum rule constraint [31], modified by the leading order QCD-correction, we obtain

$$\begin{aligned} \frac{1}{6} \left(\frac{g_A}{g_V} \right) \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) &= \int_0^1 dx \left(g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) \\ &= \int_0^1 dy \left(g_1^p(y) \right)_{\text{bag}} \int_0^1 dz \mathcal{P}_v(z, Q^2) \phi_v(z, Q^2), \end{aligned} \quad (54)$$

so that

$$\int_0^1 dz \mathcal{P}_v(z, Q^2) \phi_v(z, Q^2) = \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right), \quad (55)$$

employing (29). This relation together with the normalization (37) and the condition $|\mathcal{P}_v(z, Q^2)| \leq 1$ essentially determines the shape of the polarization function of the valence quarks. Following Carlitz and Kaur [32] we choose the parametrization

$$\mathcal{P}_v(z, Q_0^2) = \left[1 + P_v \frac{(1-z)^2}{\sqrt{z}} \right]^{-1} \quad (56)$$

at a fixed resolution scale $Q_0^2 = 10 \text{ GeV}^2$. In order to fulfill (55), P_v has to be settled at 0.035.

We now proceed to an investigation of the sea polarization. As a starting point it is rather instructive to see which shape of g_1^p would be produced under the assumption of an entirely unpolarized sea, i.e. $\mathcal{P}_s(z, Q_0^2) \equiv 0$. The corresponding result is depicted in Fig. 9. It is quite conspicuous that such a scenario leads to a sizable overestimation of g_1^p in the range $0.01 \leq x \leq 0.2$ where the sea plays an important role. This observation suggests a negative sea polarization in this range which we want to determine in the following. In order to achieve this goal, we try to reproduce the data as well as possible with the help of a parametrization for \mathcal{P}_s based on an analogy to Eq. (56). For this purpose we find the choice

$$\mathcal{P}_s(z, Q_0^2) = - \left[1 + P_s \frac{(1-z)^2}{z} \right]^{-1} \quad (57)$$

to be fairly appropriate. Fig. 10 shows the sea contribution to the polarized structure function g_1^p which is, combined with the previously discussed valence contribution,

necessary to describe the available data. This analysis can be directly transformed into the polarization function of the sea quarks, resulting in $P_s \approx 0.25$. The reliability of this parametrization is, however, restricted to the range $0.01 \leq x \leq 0.3$ since on the one hand the influence of the sea quarks is absolutely negligible for larger x so that in the range $x \geq 0.3$ the sea polarization cannot be extracted from the data anyway. Moreover, in the region $x \leq 0.01$ the SMC-data feature a rapid increase and even lie above the valence part of g_1^p , thus indicating that there may be a sign flip in the polarization of the sea quarks which would cause a breakdown of the parametrization (57). Due to the huge error bars in the very small- x region such a conclusion is, however, premature and one has to admit that there is at present no way to exactly determine the behaviour of the polarization function for $x \leq 0.01$. This is just the crucial range for the evaluation of the integral over g_1^p which sensitively depends on the extrapolation to $x = 0$. Therefore we believe that an analysis of the spin structure of the nucleon purely based on the first moment of g_1^p is rather questionable.

Taking our parametrization serious even in the limit $x \rightarrow 0$, we obtain the integral $\int_0^1 dx g_1^p(x, Q_0^2) = 0.140$ and for the total quark spin content we find

$$\begin{aligned}
\Sigma(Q_0^2) &\equiv \sum_{i=1}^{2n_f} \int_0^1 dx \Delta q_i(x, Q_0^2) \\
&= \frac{18}{5} \int_0^1 dy \left(g_1^p(y) \right)_{\text{bag}} \left(\int_0^1 dz \mathcal{P}_v(z, Q_0^2) \phi_v(z, Q_0^2) \right. \\
&\quad \left. + 5 \int_0^1 dz \mathcal{P}_s(z, Q_0^2) \phi_s(z, Q_0^2) \right) \\
&= 0.60 - 0.22 = 0.38
\end{aligned} \tag{58}$$

which is considerably higher than the original results [34, 35]. According to the spindependent evolution equations [17] this decomposition remains unchanged for all values of Q^2 . This implies the following spin structure of the proton. In the low energy regime we have the dynamics of three bag quarks whose relativistic motion intrinsically also involves orbital angular momenta so that the quark spin contribution reduces from 1 to 0.60 at this level. In a DIS experiment the virtual photon resolves the valence and sea quarks inside the constituent quarks. While the valence quarks are essentially polarized parallel to the constituent quarks, the spin alignment of the sea quarks, at least for $x \geq 0.01$, causes a partial shielding

of the spin carried by the valence quarks. The extrapolation to the very small- x regime, i.e. $x \leq 0.01$, is, however, absolutely not clear yet so that we do not want to rush to conclusions. At any rate we believe that our results for $\int_0^1 dx g_1^p(x, Q_0^2)$ and Σ give lower bounds for these quantities. Furthermore we will not specify the flavour decomposition of Σ since this clearly depends on our *assumptions* for the distribution of the strange-quarks as well as their polarization.

Of course, the formalism of the convolution model can also be applied to the polarized structure function of the neutron, g_1^n , as well so that a prediction can be made. Proceeding on the assumption of $SU(6)$ -symmetry, g_1^n is identical with the sea contribution to g_1^p and thus given by the sea part of (53). The data of SLAC [9] and SMC [10] having been taken, however, at an average Q^2 of 2 GeV^2 and 4.6 GeV^2 , respectively, a Q^2 -evolution of the spindependent quark distributions which we parametrized at $Q_0^2 = 10 \text{ GeV}^2$ is required. Like in the unpolarized case the evolution equations can be expressed in terms of the functions characterizing the substructure of the constituent quarks, with the only difference that the moments involving the sum of spin up and down polarization, Eqs. (45)–(47), have to be replaced now by the respective ones involving the difference. In order to perform the Q^2 -evolution explicitly, also the gluon polarization function is needed in addition to the previously studied polarization functions of valence and sea quarks. For lack of any better knowledge we take the polarized gluon distribution function from a perturbative QCD calculation which we are going to discuss elsewhere [36]. The resulting shape of the structure function $g_1^n(x, \bar{Q}^2 = 3.3 \text{ GeV}^2)$ at an average Q^2 between the SLAC- and SMC-data set is presented in Fig. 11, clearly demonstrating that our convolution model prediction for g_1^n is in accordance with experiment. From that one can draw the conclusion that the sea polarization based on the analysis of g_1^p is fully consistent with the data of the neutron structure function.

7 The polarized structure function g_2

A formal analysis in the framework of the operator product expansion (OPE) reveals that in contrast to g_1 the second spindependent structure function g_2 also receives twist-3 contributions [13, 18]. Accordingly we write

$$g_2(x, Q^2) = g_2^{\text{T}2}(x, Q^2) + g_2^{\text{T}3}(x, Q^2) . \quad (59)$$

The twist-2 contribution of g_2 can be directly reconstructed from g_1 , yielding [37]

$$g_2^{\text{T2}}(x, Q^2) = \int_x^1 \frac{dy}{y} g_1(y, Q^2) - g_1(x, Q^2) , \quad (60)$$

whereas the twist-3 contribution is a consequence of the quark-gluon correlations in the nucleon and with that also a reflection of confinement, thus representing very complicated physics. These twist-3 effects cannot be neglected a priori and should occur in a model which incorporates any confinement mechanism. In the bag model they are relatively large and stem from the influence of the bag surface which simulates confinement in a phenomenological way [18]. The structure function g_2 involving such complicated physics, the parton model interpretation of $g_T \equiv g_1 + g_2$, see (18), is of course rather questionable so that, being rigorous, not even a basis of parametrization is available yet.

In order to get a first impression of g_2 , however, we take a pragmatic point of view in the following and maintain the parton model relation (18) as an exemplary ansatz. According to (33) the bag model result (26) is then interpreted in terms of transversely polarized constituent quark distributions which are not identical to the longitudinally polarized quark distributions entering g_1 , but also receive modifications due to quark-gluon correlations, i.e. the simulation of confinement. Starting from the constituent quark distributions, one again can make the transition to the parton level via the convolution model approach. In order to give an estimate of g_2 , let us now assume that the twist-3 effects are fully parametrized by the bag model calculation. In this case the substructure of the constituent quarks does not change compared to the longitudinal polarization and the parton distribution functions can be taken over from the analysis of g_1 , so that g_T is just determined by (53) after replacing $(g_1(y))_{\text{bag}}$ by $(g_T(y))_{\text{bag}}$. From that we finally extract g_2 . The result is presented in Fig. 12 which separately displays the twist-2 and twist-3 contributions. The twist-3 part shows a very similar characteristic in x like the absolute value of the twist-2 part and both are definitely of the same order of magnitude. For the integral over g_2 we find the value 0.002 which is very close to the Burkhardt-Cottingham sum rule $\int_0^1 dx g_2^p(x) = 0$ [38]. There is no evidence so far that this sum rule may be violated.

Concluding the considerations of this section, we would like to emphasize once more that our previously given result of g_2 is not supposed to be a high precision

prediction, but thought as a reasonable estimate based on the convolution model. For lack of any feasible scheme for the full Q^2 -evolution of twist-3 structure functions there is also no way to predict g_2 at various Q^2 , provided it is known at a scale Q_0^2 .

8 Conclusions and outlook

In this work we presented a convolution model approach for the investigation of the nucleon structure functions. Such a model is based on the concept of effective degrees of freedom describing the low energy properties of the nucleon. In DIS, however, their fermionic substructure, characterized by the parton distribution functions ϕ_v and ϕ_s , is directly probed. These functions involve three parameters which were determined at a reference scale $Q_0^2 = 10 \text{ GeV}^2$ by experimental input. The Q^2 -evolution equations could be formulated at the level of the parton distributions ϕ , thus unravelling the substructure of the constituent quarks in dependence on the resolution scale. Comparing our results for $F_2^p(x, Q^2)$ at various Q^2 with the data, we found the picture of the convolution model to be fully consistent with experimental findings. In the case of the difference $F_2^p - F_2^n$ we observed, however, obvious discrepancies in the large- as well as in the small- x region which can be attributed to the use of $SU(6)$ -symmetry at the constituent quark level and isospin symmetry of the sea quarks, respectively.

Subsequent to the investigation of the unpolarized structure functions we applied the formalism of the convolution model to the spindependent structure functions $g_1^{p,n}$. In addition to the nonperturbative information of the bag model calculation and the distribution functions ϕ the expressions for $g_1^{p,n}$ also involve polarization functions of the partons. An analysis of the x -dependence of the proton data taken so far implied that the valence quarks are predominantly polarized parallel to the constituent quarks while the sea quarks tend to screen the spin contribution of the valence quarks. Having extrapolated our parametrization of the sea polarization function, which can be pinned down rather well for $0.01 \leq x \leq 0.2$, to $x = 0$, we obtained the quark spin decomposition $\Sigma = \Sigma_v + \Sigma_s = 0.60 - 0.22 = 0.38$. We would like to emphasize, however, that the small x -behaviour of the sea polarization, particularly the one for $x \leq 0.01$, and accordingly also the integrated spin content of the sea quarks remain essentially undetermined yet. For this reason one definitely

cannot exclude the possibility of a totally unpolarized sea, i.e. $\Sigma_s = 0$, at the moment. With the polarization functions in hands we could finally make a prediction for g_1^n . Comparison with experiment indicated that the sea contribution to the proton structure function is absolutely consistent with the full neutron structure function. From that one can also conclude that the proton and neutron data together are perfectly compatible with the fundamental Bjorken sum rule which is fulfilled in our model per construction.

In the last section of our work we presented an estimate for the transversely polarized structure function g_2 . In doing this we took the naive parton model interpretation as a reasonable guideline and assumed that the twist-3 effects are fully parametrized by the confinement mechanism of the bag model. The resulting estimate for g_2^p in the convolution model suggests that twist-3 contributions may be relatively important even for values of Q^2 of the order of 10 GeV^2 .

As became clear in the course of our discussions there are some points which call for further investigations. The first problem is associated with the small- x region of $F_2^p - F_2^n$ and consequently connected with the deviation of the GSR from $1/3$. Here we aim at a microscopic mechanism which explains the preference of d-quarks over u-quarks in the proton sea. This requires an extension of the convolution model which in addition to the three constituent quarks also includes a meson cloud. Another challenge consists in achieving a microscopic understanding of the sea polarization as given by our analysis. The spin problem would then finally loose its magic. Especially this issue is intensively worked on [36].

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Figure captions

Fig. 1. Deep inelastic lepton–nucleon scattering in the one photon exchange approximation.

Fig. 2. DIS in the convolution model for the nucleon substructure.

Fig. 3. The unpolarized structure function $F_2^p(x, Q_0^2 = 10 \text{ GeV}^2)$ in the convolution model. The data are taken from [26].

Fig. 4. Q^2 –evolution of $\phi_v(z, Q^2)$.

Fig. 5. Q^2 –evolution of $\phi_s(z, Q^2)$.

Fig. 6. F_2^p evolved to $Q^2 = 5 \text{ GeV}^2$. The data are taken from [28].

Fig. 7. F_2^p evolved to $Q^2 = 50 \text{ GeV}^2$. The data are taken from [26].

Fig. 8. $[F_2^p - F_2^n](x, Q^2 = 40 \text{ GeV}^2)$ under the assumption of an isospin symmetric sea. The data are taken from [29].

Fig. 9. $g_1^p(x, Q_0^2 = 10 \text{ GeV}^2)$ in the convolution model under the assumption of an identically vanishing sea polarization. The data are taken from [7, 8, 33].

Fig. 10. Valence and sea contribution to the structure function $g_1^p(x, Q_0^2 = 10 \text{ GeV}^2)$. The data are taken from [7, 8, 33].

Fig. 11. The neutron structure function $g_1^n(x, \bar{Q}^2)$ at an average $\bar{Q}^2 = 3.3 \text{ GeV}^2$ in comparison with the data [9, 10].

Fig. 12. An estimate for $g_2^p(x, Q_0^2 = 10 \text{ GeV}^2)$ in the convolution model.

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